

2.1 Derivative Basics: Form J Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_FJ.py`). All problems are original; provenance is in `PROVENANCE_GJ.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Answer-only items** (Q1, Q9): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse), or a number from the wrong column of a table, earns 0 for that part unless a row below says otherwise.
- **Show-work items** (Parts II–IV other than Q9, and Part VI): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty.

Form J

Item	Answer	Rubric / trap
1(a)	4	<i>Trap:</i> the tangent drawn there invites the slope, 2; earns 0.
1(b)	-2	<i>Trap:</i> giving the height $f(-5) = 4$; earns 0.
1(c)	0	<i>Trap:</i> giving the height 2 of the low point; earns 0.
1(d)	DNE	Corner at $(-1, 4)$: one-sided slopes 2 and -2 . <i>Trap:</i> 0, because $(-1, 4)$ is a high point.
1(e)	$(-3, -1)$ and $(1, 4)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap:</i> $(-3, 4)$, which includes the corner and the falling stretch $(-1, 1)$.
1(f)	$f'(-5) < 0 < f'(3)$	$f'(-5) = -2$, $f'(3) = 2$. 3 pts, all or nothing.
2	$y + 2 = 4(x - 3)$, i.e. $y = 4x - 14$	2 pts point $(3, -2)$ and slope 4 in point-slope form; 3 pts equation. <i>Trap:</i> there is no formula to differentiate, and students hunt for one. Swapping the value and the slope, or a sign slip with $x - (3)$, earns 2.
3	$y - 1 = 6(x - 2)$, i.e. $y = 6x - 11$	1 pt $f(2) = 1$; 2 pts $f'(x)$; 1 pt $f'(2) = 6$; 1 pt equation in any correct form. <i>Trap:</i> $f(2) = 8 - 4 - 4 + 1 = 1$ and $f'(x) = 3x^2 - 2x - 2$, $f'(2) = 12 - 4 - 2 = 6$. Using $f(2) = 1$ as the slope earns 2 of 5.
4(a)	1	3 pts. Simplifies to $y = x + 7$. <i>Trap:</i> 6 (before dividing by 3) or 2; earns 0.
4(b)	5	2 pts. Substitute $x = -2$. <i>Trap:</i> 3 or 0, from reading pieces of the equation; earns 0.
5	2	<i>Trap:</i> u' is printed and invites $u'(2) = 9$; earns 0.
6	$f'(x) = \frac{4}{(4x+1)^2}$	2 pts $f(x+h) = -\frac{1}{4(x+h)+1} = -\frac{1}{4x+4h+1}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $-(4x+1) + (4x+4h+1) = 4h$; 2 pts cancel h to get $\frac{4}{(4x+4h+1)(4x+1)}$; 2 pts let $h \rightarrow 0$. <i>Trap:</i> writing $f(x) + h$ for $f(x+h)$, or losing a sign across the subtraction. Rules only: at most 2.

Item	Answer	Rubric / trap
7	8	1 pt $g(-2) = -7$; 1 pt setup; 3 pts factoring out $x+2$ to get x^2-x+2 , by any valid method. Synthetic or long division by $x+2$ works (on 1, 1, 0, 4, with the zero placeholder). So does grouping: $x^3+x^2+4 = (x^3+8)+(x^2-4) = (x+2)(x^2-2x+4) + (x+2)(x-2) = (x+2)(x^2-x+2)$. No particular method is required. 3 pts value. <i>Trap</i> : $g(-2) = -8+4-3 = -7$, so subtracting it gives $+7$ and the numerator is x^3+x^2+4 ; the denominator is $x+2$. A student who drops the $0x$ placeholder gets a wrong quotient. Rules only: at most 2.
8(a)	0	$-1 = \sin \frac{3\pi}{2}$, so this is the derivative of $\sin$ at $\frac{3\pi}{2}$: $\cos \frac{3\pi}{2} = 0$. <i>Trap</i> : “0/0, DNE” or -1 ; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
8(b)	$-\frac{1}{9}$	the derivative of $\frac{1}{x}$ at 3: $-\frac{1}{9}$; a common denominator also works. <i>Trap</i> : $\frac{1}{9}$ (sign) or “0/0”; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
9	(a) 0 (b) 0	3 each. <i>Traps</i> : (a) $3\cos(\pi) = -3$; (b) $f'(-2) = -7$. In (b), $m(x)$ is the same number for every x , so $m'(x) = 0$; the problem says “for every x ”.
10(a)	$-2x^{-2} - \frac{5}{3}x^{-2/3}$	1 pt rewriting $2x^{-1} - 5x^{1/3}$; 2 pts derivative. <i>Trap</i> : $\frac{1}{3} - 1 = -\frac{2}{3}$; the derivative of $2x^{-1}$ is $-2x^{-2}$.
10(b)	$3x^2$	1 pt rewriting x^3+2 (for $x \neq 0$); 2 pts derivative. <i>Trap</i> : $\frac{5x^6}{5x^3} = x^3$, not x^2 ; differentiating top and bottom separately earns 0.
10(c)	$18x - 12 - 6\sin x$	1 pt rewriting $9x^2 - 12x + 4 + 6\cos x$; 2 pts derivative. <i>Trap</i> : $(3x-2)^2 \neq 9x^2 + 4$; the derivative of $6\cos x$ is $-6\sin x$.
10(d)	$\frac{3}{2}x^{1/2} - \frac{9}{2}x^{-1/2}$	1 pt rewriting $x^{3/2} - 9x^{1/2}$; 2 pts derivative. <i>Trap</i> : $\sqrt{x} \cdot x = x^{3/2}$; $9 \cdot \frac{1}{2} = \frac{9}{2}$.
11	$3 + \frac{\pi^2}{3}$	3 pts $f'(x) = 6\cos x + 3x^2$; 2 pts value. <i>Trap</i> : $\cos \frac{\pi}{3} = \frac{1}{2}$; $3\left(\frac{\pi}{3}\right)^2 = \frac{\pi^2}{3}$. Using $\sin \frac{\pi}{3}$ gives $3\sqrt{3} + \frac{\pi^2}{3}$, which earns 3.
12	$x = \frac{4\pi}{3}$	2 pts $f'(x) = -2\sin x - \sqrt{3}$; 2 pts $\sin x = -\frac{\sqrt{3}}{2}$; 2 pts the solutions in the interval. <i>Trap</i> : $\frac{5\pi}{3}$ also solves $\sin x = -\frac{\sqrt{3}}{2}$ but lies outside $[\pi, \frac{3\pi}{2})$; listing it costs 2.
13	II	6 pts, all or nothing. In II the dashed curve is $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x$ and the solid curve is $f'(x) = x^2 - x - 2$: it is zero exactly where the dashed curve turns ($x = -1, 2$), positive where that curve rises, and negative where it falls. Wrong choices: I: right zeros, wrong sign: it is negative where the cubic rises (left of $x = -1$); III: shifted: the other curve is zero at $x = -2$ and $x = 1$, not at the turning points $x = -1$ and $x = 2$; IV: the line is the derivative of the derivative: it is zero at $x = \frac{1}{2}$, not where the cubic turns. <i>Trap</i> : assuming the dashed curve is always the derivative; the styles swap between graphs.
14(a)	4	$h'(x) = 4f'(x) - g'(x) + 2x$, so $h'(-1) = 4(1) - (-2) + (-2)$. 1 pt rule, 2 pts correct table entries, 1 pt value. <i>Trap</i> : the columns run $x, f(x), g(x), f'(x), g'(x)$, not the usual order. Using the value columns gives 6, which is $h(-1)$; earns 1 (for the rule, if shown). Forgetting the x^2 term's derivative gives 6.
14(b)	-16	$f(0) = -3$ is a number, so $k(x) = -3x^2 - g(x)$ and $k'(x) = -6x - g'(x)$; $k'(2) = -12 - (4) = -16$. 2 pts $k'(x)$, 2 pts value. <i>Trap</i> : using $f'(0) = 2$ gives 4; earns at most 1.
14(c)	$x = 2$ only	$(f+g)' = f' + g'$, which is $-1, 2, 0, 4$ at $x = -1, 0, 2, 5$. 2 pts for using $f' + g'$, 2 pts for the answer. <i>Trap</i> : $x = 5$, where $f(x) + g(x) = 0$: there the graph crosses the x -axis; it is not flat. Earns 0.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 6 + 6 + 12 + 5 + 6 + 6 + 12 = 100$.